ON THE PERTURBATIVE SOLUTIONS OF BOHMIAN QUANTUM GRAVITY

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In this paper we have solved the Bohmian equations of quantum gravity, perturbatively. Solutions up to second order are derived explicitly, but in principle the method can be used in any order. Some consequences of the solution are disscused.

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I. INTRODUCTION

Recently [1–3], a perturbative method for solving classical Einstein's equations in its Hamilton–Jacobi form is presented. The method rests on expanding the Hamilton–Jacobi generating functional in terms of the powers of spatial gradients of the metric and matter fields, and then solving the equations order by order. This expansion is valid when the characteristic scale of spatial variation of physical quantities is larger than the characteristic lenght of the theory, e.g. the Hubble's radius. In fact it can be shown that the solution can be calculated at any order. The form of the Hamilton–Jacobi generating functional in each order is chosen such that it be 3-diffeomorphic invariant.

This method is used and examined for many physical cases. Salopek et. al. [1] have solved the Hamilton–Jacobi and the momentum constraint equations in the presence of matter fields up to second order in spatial gradients. Parry et. al. [2] have used a specific conformal transformation of 3–metric to simplify the Hamiltonian and solved the problem in higher orders of spatial gradients. Then they have compared their results with exact solutions for some specific cases and obtained a recursion relation for different orders and so they have presented the solution up to any order. In addition similar calculations are made for Brans–Dicke theory. [3]

An essential question would be can the method be applied to quantum gravity realm. Unfortunately there are different approaches to quantum gravity, non of them completely acceptable and self consistent. These include, the standard Wheeler–De Witt canonical approach [4], the Hawking path integral approach [5], the Narlikar–Padmanabhan quantization of conformal degree of freedom of the space–time metric [6], the Bohmian approach to quantum gravity [7] and the approach presented by author et.al. as geometrization of quantum theory [8]. Among these approaches Bohmian quantum gravity is of our concern here, becuase it highly relates to Hamilton–Jacobi theory. In fact as we shall review in the next section, in Bohmian quantum gravity one encounters with a modified Hamilton–Jacobi equation.

We shall apply the above–mentioned perturbative method for solving Bohmian quantum gravity equations. We shall do this up to the second order, but in principle the metod can be applied to any order.

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II. BOHMIAN QUANTUM GRAVITY

Bohm's theory is a causal version of quantum mechanics [9]. According to this theory, any particle is acompanied with an objectively real field (Ψ) satisfying Schrödinger equation. This field exerts a quantum force derivable from a quantum potential given by

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2|\Psi|}{|\Psi|} \tag{1}$$

This theory is motivated from the fact that when one sets $\Psi = |\Psi|\exp[i\mathcal{S}/\hbar]$ in the Schrödinger equation, one arrives at a continuity equation:

$$\frac{\partial|\Psi|^2}{\partial t} + \vec{\nabla} \cdot \left(|\Psi|^2\frac{\vec{\nabla}\mathcal{S}}{m}\right) = 0 \tag{2}$$

and a modified Hamilton–Jacobi equation:

$$\frac{\partial\mathcal{S}}{\partial t} + \frac{|\vec{\nabla}\mathcal{S}|^2}{2m} + V + Q = 0 \tag{3}$$

in which V represents the classical potential and Q, the quantum potential is defined as above. It is the main positive point of Bohm's theory which using only quantum potential is able to explain all enigmatic aspects of quantum theory. These includes presentation of a causal description for wave–function collapse during a measurement, and description of uncertainty relations and also presentation of particle trajectories [9]. Particle trajectory can be obtained through the modified Hamilton–Jacobi equation (3) and using the guidance relation $\vec{p} = \vec{\nabla}\mathcal{S}$, or using the Newton's law of motion including the quantum potential. It is worth noting that the trajectories explain many nonordinary behaviour

in quantum mechanics. For example particle trajectories in a two–slit experiment can be calculated [7] and it can be seen how quantum potential forces particles to move in such a way to make the interference pattern.

Bohm's theory can be applied to any system. Application of this theory to gravity leads to Bohmian quantum gravity. Its properties and positive points are expressed in the literature [7,9]. Application of Bohm's theory to quantum gravity has several advantages. First of all in this approach, different quantities like the 3–space geometry, intrinsic and extrinsic curvatures of the space–like surfaces and so on have physical reality without any dependence upon the measurment process. Second, the metric has a definite time evolution in this theory. Third, in this approach the wave function has two roles. One role in generating the quantum potential and another as the probabilistic interpretation. When one deals with a single system (as is the case for quantum cosmology) for which the probability is not defined, the first role of the wave function is important. Note that in the standard quantum theory, only the second role is highlighted and thus the meaning of the wave function is questionable in quantum gravity. Finally, the classical limit is well defined in Bohm's theory. When both quantum potential and its gradiant are small compared to the classical potential and its gradiant, then we are in the classical limit. This allows one, in specific cases, to have quantum effects at large scales and classical limit in small scales.

Here we use Bohmian quantum gravity, not only because of the above mentioned advantages, but also because it highly relates to the Hamilton–Jacobi equation. Before

proceeding, we present the Bohmain equations for quantum gravity [7] which we shall refer to later. These equations are[1] (setting $\hbar = c = 8\pi G = 1$):

$$\frac{\delta}{\delta h_{ij}}\left(2h^q G_{ijkl}\frac{\delta \mathcal{S}}{\delta h_{kl}}\mathcal{A}^2\right) + \frac{\delta}{\delta \phi}\left(\frac{h^q}{\sqrt{h}}\frac{\delta \mathcal{S}}{\delta \phi}\mathcal{A}^2\right) = 0 \tag{4}$$

$$G_{ijkl}\frac{\delta \mathcal{S}}{\delta h_{ij}}\frac{\delta \mathcal{S}}{\delta h_{kl}} + \frac{1}{2\sqrt{h}}\left(\frac{\delta \mathcal{S}}{\delta \phi}\right)^2 - \sqrt{h}\left(\mathcal{R}^{(3)} + 2\Lambda - Q_G\right) + \frac{1}{2}\sqrt{h}h^{ij}\partial_i\phi\partial_j\phi + \frac{1}{2}\sqrt{h}(V + Q_M) = 0$$

$$\tag{5}$$

$$Q_G = -\frac{1}{\sqrt{h}\mathcal{A}}\left(G_{ijkl}\frac{\delta^2 \mathcal{A}}{\delta h_{ij}\delta h_{kl}} + h^{-q}\frac{\delta h^q G_{ijkl}}{\delta h_{ij}}\frac{\delta \mathcal{A}}{\delta h_{kl}}\right) \tag{6}$$

$$Q_M = -\frac{1}{h\mathcal{A}}\frac{\delta^2 \mathcal{A}}{\delta \phi^2} \tag{7}$$

$$2\nabla_j\frac{\delta \mathcal{A}}{\delta h_{ij}} - h^{ij}\partial_j\phi\frac{\delta \mathcal{A}}{\delta \phi} = 0 \tag{8}$$

$$2\nabla_j\frac{\delta \mathcal{S}}{\delta h_{ij}} - h^{ij}\partial_j\phi\frac{\delta \mathcal{S}}{\delta \phi} = 0 \tag{9}$$

in which $\mathcal{A}$ is the norm of the wave function, $\mathcal{S}$ is its phase times $\hbar$ and is in fact the quantum Einstein–Hamilton–Jacobi function, q is an ordering parameter, h_{ij} is the spatial metric in ADM decomposition of the space–time metric, G_{ijkl} is super metric on 3–space, ϕ denotes the matter field, and Q_G and Q_M are gravity and matter quantum potentials respectively.

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[1]They can be obtained by setting $\Psi = \mathcal{A}\exp[i\mathcal{S}/\hbar]$ in the WDW equation and the 3–diffeomorphism invariance condition.

Equation (4) is the continuity equation representing the conservation law of probability in the super space, and equation (5) is the quantum Einstein–Hamilton–Jacobi equation, which shows that the difference between quantum and classical worlds is only the presence of the quantum potential consisting of two terms, gravity and matter quantum potentials. Equations (8) and (9) are 3-diffeomorphism invariance conditions for $\mathcal{A}$ and $\mathcal{S}$. Time evolution of metric and the matter field can be derived from the canonical relations:

$$\pi^{kl} = \frac{\delta \mathcal{S}}{\delta h_{kl}} = \frac{\sqrt{h}}{2}(K^{kl} - h^{kl}K) \tag{10}$$

$$\pi_\phi = \frac{\delta \mathcal{S}}{\delta \phi} = \frac{\sqrt{h}}{N}\dot{\phi} - \sqrt{h}\frac{N^i}{N}\partial_i\phi \tag{11}$$

$$K_{ij} = \frac{1}{2N}(\nabla_i N_j + \nabla_j N_i - \dot{h}_{ij}) \tag{12}$$

in which N and N^i are the lapse and shift functions respectively, and K_{ij} is the extrinsic curvature of the 3–space. It can be seen that in Bohmian quantum gravity, there is no time problem. Time emerges naturally from the equations of motion. Bohmian trajectories can be obtained from the above equations. For example, the Bohmian trajectories for Rabertson–Walker universe are derived by Horiguchi in [7] and other references cited in [7]. Another example is Bohmian trajectories for black holes. They are obtained in [10]. In this reference it is shown that the quantum black hole geometry is highly sensible to the ordering parameter. For some specific ordering parameter, Bohmian quantum gravity presents a good framework for understanding Hawking radiation. Some other aspects of Bohmian quantum gravity can be found in [11]. For a complete review of the theory see [7].

III. SOLVING THE EQUATIONS

It is disscussed in the previous section that the complete set of equations of quantum gravity are equations (4), (5), (8), and (9). The first is the continuity equation, while the second is the quantum Einstein–Hamilton–Jacobi equation. The third and fourth equations gurantee that $\mathcal{A}$ and $\mathcal{S}$ be 3-diffeomorphic invariants. A perturbative solution can be achieved via expansion of $\mathcal{S}$ and $\mathcal{A}$ in terms of powers of spatial gradients. In the long–wavelength approaximation a few terms of the expansion is sufficient. Therefore one should set:

$$\Omega = \sum_{n=0}^{\infty} \Omega^{(2n)}; \quad \mathcal{A} = e^{\Omega} \tag{13}$$

$$\mathcal{S} = \sum_{n=0}^{\infty} \mathcal{S}^{(2n)} \tag{14}$$

Note that introducing the new functional Ω will simplifies the equations. In each order, the two coupled equations quantum Einstein–Hamilton–Jacobi equation and continuity equation should be solved. The two other equations only show that the functionals $\mathcal{S}^{(2n)}$ and $\Omega^{(2n)}$ must be 3-diffeomorphic invariants. By considering special forms for $\mathcal{S}^{(2n)}$ and $\Omega^{(2n)}$, these equations would be satisfied automatically.

A. Zeroth Order Solution

In this order, the continuity equation reads as:

$$-\left(q + \frac{3}{2}\right) h_{ij} \frac{\delta \mathcal{S}^{(0)}}{\delta h_{ij}} + 4\sqrt{h} G_{ijkl} \frac{\delta \Omega^{(0)}}{\delta h_{ij}} \frac{\delta \mathcal{S}^{(0)}}{\delta h_{kl}}$$

$$+ 2\sqrt{h}G_{ijkl}\frac{\delta^2\mathcal{S}^{(0)}}{\delta h_{ij}\delta h_{kl}} + 2\frac{\delta\Omega^{(0)}}{\delta\phi}\frac{\delta\mathcal{S}^{(0)}}{\delta\phi} + \frac{\delta^2\mathcal{S}^{(0)}}{\delta\phi^2} = 0 \tag{15}$$

while the zeroth order quantum Einstein–Hamilton–Jacobi equation is:

$$2\sqrt{h}G_{ijkl}\frac{\delta\mathcal{S}^{(0)}}{\delta h_{ij}}\frac{\delta\mathcal{S}^{(0)}}{\delta h_{kl}} + \left(\frac{\delta\mathcal{S}^{(0)}}{\delta\phi}\right)^2 - 2\sqrt{h}G_{ijkl}\frac{\delta^2\Omega^{(0)}}{\delta h_{ij}\delta h_{kl}}$$

$$- 2\sqrt{h}G_{ijkl}\frac{\delta\Omega^{(0)}}{\delta h_{ij}}\frac{\delta\Omega^{(0)}}{\delta h_{kl}} + \left(q + \frac{3}{2}\right)h_{ij}\frac{\delta\Omega^{(0)}}{\delta h_{ij}} - \frac{\delta^2\Omega^{(0)}}{\delta\phi^2} - \left(\frac{\delta\Omega^{(0)}}{\delta\phi}\right)^2 = 0 \tag{16}$$

in which for simplicity of calculations, we have assumed that the scalar field has no self interaction, i.e we have set $V(\phi) = 0$.

In order to $\mathcal{S}^{(0)}$ and $\Omega^{(0)}$ be 3-diffeomorphic invariants and thus satisfy equations (8) and (9) automatically, one should set:

$$\mathcal{S}^{(0)} = \int d^3x\sqrt{h}H(\phi) \tag{17}$$

$$\Omega^{(0)} = \int d^3x\sqrt{h}K(\phi) \tag{18}$$

in which H and K are functions of the scalar field and contain no spatial derivatives. Since $d^3x\sqrt{h}$ is 3-diffeomorphic invariant measure, the above experssions are also 3-diffeomorphic invariant. By substituting these relations for $\Omega^{(0)}$ and $\mathcal{S}^{(0)}$ in equations (15) and (16), we have the following equations for H and K:

$$\frac{d^2H}{d\phi^2} - \frac{3}{2}(q+5)H + 2\sqrt{h}\left(\frac{dH}{d\phi}\frac{dK}{d\phi} - \frac{3}{4}KH\right) = 0 \tag{19}$$

$$\frac{d^2K}{d\phi^2} - \frac{3}{2}(q+5)K - \sqrt{h}\left(\left(\frac{dH}{d\phi}\right)^2 - \left(\frac{dK}{d\phi}\right)^2 + \frac{3}{4}K^2 - \frac{3}{4}H^2\right) = 0 \tag{20}$$

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Setting both metric–dependent (terms containing $\sqrt{h}$) and metric–independent terms equal to zero, we have four equations with the simultaneous solution:

$$H = Ae^{\alpha\phi}; \quad \alpha = \pm\frac{\sqrt{3}}{2} \tag{21}$$

$$K = BH \tag{22}$$

$$q = -\frac{9}{2} \tag{23}$$

in wich A and B are constants of integration. It must be noted here that using this solution it is a simple task to show that quantum potential is zero at this order. So the solution at this order is in fact classical.

B. Second Order Solution

In the second order, the continuity and quantum Einstein–Hamilton–Jacobi equations are respectively:

$$-\left(q+\frac{3}{2}\right)h_{ij}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{ij}} + 4\sqrt{h}G_{ijkl}\frac{\delta\Omega^{(0)}}{\delta h_{ij}}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{kl}} + 4\sqrt{h}G_{ijkl}\frac{\delta\Omega^{(2)}}{\delta h_{ij}}\frac{\delta\mathcal{S}^{(0)}}{\delta h_{kl}}$$

$$+ 2\sqrt{h}G_{ijkl}\frac{\delta^2\mathcal{S}^{(2)}}{\delta h_{ij}\delta h_{kl}} + 2\frac{\delta\Omega^{(0)}}{\delta\phi}\frac{\delta\mathcal{S}^{(2)}}{\delta\phi} + 2\frac{\delta\Omega^{(2)}}{\delta\phi}\frac{\delta\mathcal{S}^{(0)}}{\delta\phi} + \frac{\delta^2\mathcal{S}^{(2)}}{\delta\phi^2} = 0 \tag{24}$$

$$2\sqrt{h}G_{ijkl}\frac{\delta\mathcal{S}^{(0)}}{\delta h_{ij}}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{kl}} + \frac{\delta\mathcal{S}^{(0)}}{\delta\phi}\frac{\delta\mathcal{S}^{(2)}}{\delta\phi} - \sqrt{h}G_{ijkl}\frac{\delta^2\Omega^{(2)}}{\delta h_{ij}\delta h_{kl}} - 2\sqrt{h}G_{ijkl}\frac{\delta\Omega^{(0)}}{\delta h_{ij}}\frac{\delta\Omega^{(2)}}{\delta h_{kl}}$$

$$+ \frac{1}{2}(q+3)h_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} - \frac{1}{2}\frac{\delta^2\Omega^{(2)}}{\delta\phi^2} - \frac{\delta\Omega^{(0)}}{\delta\phi}\frac{\delta\Omega^{(2)}}{\delta\phi} - \sqrt{h}\left(\mathcal{R}^{(3)} - \frac{1}{2}\nabla_i\phi\nabla^i\phi\right) = 0 \tag{25}$$

On using the zeroth order solution and again setting both terms with and without $\sqrt{h}$ equal to zero, one arrives at the following four equations:

$$I[\mathcal{S}^{(2)}] \equiv 2\sqrt{h}G_{ijkl}\frac{\delta^2\mathcal{S}^{(2)}}{\delta h_{ij}\delta h_{kl}} + 3h_{ij}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{ij}} + \frac{\delta^2\mathcal{S}^{(2)}}{\delta\phi^2} = 0 \tag{26}$$

$$I\!\!I[\mathcal{S}^{(2)}, \Omega^{(2)}] \equiv 2\alpha B\frac{\delta\mathcal{S}^{(2)}}{\delta\phi} - Bh_{ij}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{ij}} + 2\alpha\frac{\delta\Omega^{(2)}}{\delta\phi} - h_{ij}\frac{\delta\Omega^{(2)}}{\delta\phi} = 0 \tag{27}$$

$$I\!\!I\!\!I[\mathcal{S}^{(2)}, \Omega^{(2)}] \equiv Hh_{ij}\frac{\delta\mathcal{S}^{(2)}}{\delta h_{ij}} - 2\alpha H\frac{\delta\mathcal{S}^{(2)}}{\delta\phi}$$

$$- BHh_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} + 2\alpha HB\frac{\delta\Omega^{(2)}}{\delta\phi} + 2\sqrt{h}\left(\mathcal{R}^{(3)} - \frac{1}{2}\nabla_i\phi\nabla^i\phi\right) = 0 \tag{28}$$

$$I\!\!V[\Omega^{(2)}] \equiv 2\sqrt{h}G_{ijkl}\frac{\delta^2\Omega^{(2)}}{\delta h_{ij}\delta h_{kl}} + 3h_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} + \frac{\delta^2\Omega^{(2)}}{\delta\phi^2} = 0 \tag{29}$$

In order to solve the above equations, we use a different method with respect to the zeroth order. Our goal is to find the quantum corrections on the Hamilton–Jacobi functional at second order, to the classical functional $\mathcal{S}_c^{(2)}$. It must be noted that $I\!\!I\!\!I[\mathcal{S}^{(2)},\Omega^{(2)}] = 0$ is just the classical Einstein–Hamilton–Jacobi equation except for its third and fourth terms. So, with a glance at the form of the third and fourth terms, one easily can solve $I\!\!I\!\!I = 0$ as:

$$\mathcal{S}^{(2)} - B\Omega^{(2)} = \mathcal{S}_c^{(2)} \tag{30}$$

Therefore for finding $\mathcal{S}^{(2)}$, it is sufficient to solve $I\!\!V[\Omega^{(2)}] = 0$ to find $\Omega^{(2)}$ and use the above equation. On the other hand, since for the classical limit the $\sqrt{h}$–independent terms of the continuity equation leads to $I[\mathcal{S}_c^{(2)}] = 0$ and since I is linear, we have:

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$$I[\mathcal{S}^{(2)}] = I[\mathcal{S}_c^{(2)} + B\Omega^{(2)}] = I[\mathcal{S}_c^{(2)}] + BI[\Omega^{(2)}] = BI[\Omega^{(2)}] = BN[\Omega^{(2)}] \tag{31}$$

so

$$I[\mathcal{S}^{(2)}] = 0 \iff N[\Omega^{(2)}] = 0 \tag{32}$$

It remains for the second equation $I\!I[\mathcal{S}^{(2)}, \Omega^{(2)}] = 0$. On using the relation (30), and linearity of $I\!I$, one arrives at:

$$2\alpha \frac{\delta\Omega^{(2)}}{\delta\phi} - h_{ij}\frac{\delta\Omega^{(2)}}{\delta h_{ij}} = \frac{B}{1 + B^2}\left(-2\alpha\frac{\delta\mathcal{S}_c^{(2)}}{\delta\phi} + h_{ij}\frac{\delta\mathcal{S}_c^{(2)}}{\delta h_{ij}}\right) \tag{33}$$

which has the solution:

$$\Omega^{(2)} = -\frac{B}{1 + B^2}\mathcal{S}_c^{(2)} + \Lambda \tag{34}$$

where the functional Λ satisfies the equation:

$$2\alpha\frac{\delta\Lambda}{\delta\phi} = h_{ij}\frac{\delta\Lambda}{\delta h_{ij}} \tag{35}$$

In addition, using the relation (34) and $N[\Omega^{(2)}] = 0$ and the linearity of N one has $N[\Lambda] = 0$. So it is sufficient to find the simultaneous solution of the relations (35) and $N[\Lambda] = 0$. In finding the solution, we use the techniques of [2]. Making the conformal transformation:

$$f_{ij}(x) = F^{-2}[\phi(x)]h_{ij}(x) \tag{36}$$

one can see that the equation (35) requires F to satisfy the relation:

$$-4\alpha\frac{dF}{d\phi} = F \tag{37}$$

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with the solution $F = constant \times \exp[-\phi/4\alpha]$. The most general form of Λ is

$$\Lambda = \int d^3x \sqrt{f} \left[L(\phi)\tilde{\mathcal{R}}^{(3)} + M(\phi)\tilde{\nabla}_i\phi\tilde{\nabla}^i\phi \right] \tag{38}$$

where a tilde over any quantity represents that it is calculated using the f_{ij} metric. $\tilde{\mathcal{R}}^{(3)}$ is the Ricci scalar curvature of f_{ij}, L and M are some functions of the scalar field. The above expression is the most general form to make Λ, 3–diffeomorphic invariant and contains terms with spatial gradiants of order two. Note that terms like $\tilde{\nabla}^2\phi$ can be transformed to $\tilde{\nabla}_i\phi\tilde{\nabla}^i\phi$ by integration by part.

Now the equations (35) and $N[\Lambda] = 0$ can be solved for L and M. The solution can be transformed back to the original metric h_{ij} using the inverse of the above conformal transformation. The result is:

$$\Lambda = C \int d^3x \sqrt{h} e^{\phi/4\alpha} \left[\mathcal{R}^{(3)} - \frac{1}{6}\nabla_i\phi\nabla^i\phi \right] \tag{39}$$

where C is a constant.

For writting down $\mathcal{S}^{(2)}$ and $\Omega^{(2)}$ it is neccesary to know $\mathcal{S}_c^{(2)}$. From [2], we have:

$$\mathcal{S}_c^{(2)} = \frac{3}{10} \int d^3x \sqrt{h} e^{\alpha\phi} \left[\mathcal{R}^{(3)} - \nabla_i\phi\nabla^i\phi \right] \tag{40}$$

Thus we have:

$$\mathcal{S} = \mathcal{S}^{(0)} + \mathcal{S}^{(2)} + \cdots = \int d^3x \sqrt{h} \left\{ e^{\alpha\phi} \left[A + \frac{3}{10(1+B^2)} \left(\mathcal{R}^{(3)} - \nabla_i\phi\nabla^i\phi \right) \right] \right.$$

$$\left. + BC e^{\phi/4\alpha} \left(\mathcal{R}^{(3)} - \frac{1}{6}\nabla_i\phi\nabla^i\phi \right) \right\} \cdots \tag{41}$$

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$$\Omega = \Omega^{(0)} + \Omega^{(2)} + \cdots = \int d^3x \sqrt{h} \left\{ e^{\alpha\phi} \left[AB - \frac{3B}{10(1+B^2)} \left(\mathcal{R}^{(3)} - \nabla_i \phi \nabla^i \phi \right) \right] \right.$$

$$\left. + Ce^{\phi/4\alpha} \left(\mathcal{R}^{(3)} - \frac{1}{6} \nabla_i \phi \nabla^i \phi \right) \right\} \cdots \tag{42}$$

It is worth noting that the first two terms in $\mathcal{S}$ which are scaled by $e^{\alpha\phi}$ are of the same form as the classical solution up to second order. In fact quantum effects are introduced via the third term and the renormalization of the factor $3/10$ in the second term to $3/10(1+B^2)$. An important property of the solution is the factor $e^{4\phi/\alpha}$ of the third term which differs from the factor of the first two terms. The presence of the third term leads to new couplings between the matter field and the metric in Bohmian equations of motion, leading to highly quantic solutions.

IV. CONCLUDING REMARKS

As we saw, the Bohmian equations of motion for quantum gravity, i.e. quantum Einstein–Hamilton–Jacobi equation and continuity equation can be solved in principle as an expansion with respect to spatial gradiants. We derived the solution up to the second order. As a result since our solution contains spatial gradiants, it is useful for disscusing inhomogeneous space–times such as black holes which are partly disscused in the framework of Bohmian quantum gravity [10]. In a forthcomming paper we shall apply the result to black holes.

A point must be noted here. As we asserted previousely, according to Bohm's theory, in the classical limit quantum potential and its gradiant are small compared to classical

potential and its gradiant. This can be achieved both in the case where the norm of the wave function varies slowly and in the case where it varied highly. This is because of the fact that quantum potential is proportional to the fraction of second derivaties of the norm of the wave function and the norm itself. (see e.g. [7]). As a result, classical limit and long wavelenght limit (i.e. considering only a few terms in the expansion with respect to spatial gradiants) are not the same. So in Bohmian quantum gravity comparison of characteristic lenght of fluctuations with theory's characteristic lenght (e.g. Hubble's radius) does not lead us to anything about the fact that the limit is either classic or quantum.

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